Chapter 3
Concentration Risk in Credit Portfolios and Its Treatment Under Basel II

3.1 Types of Concentration Risk

Concentration risk can be defined as "any single exposure or group of exposures with the potential to produce losses large enough (relative to a bank's capital, total assets, or overall risk level) to threaten a bank's health or ability to maintain its core operations".[126] There are several types of concentration that can incorporate significant risks (see Fig. 3.1).

In a bank's assets there can be concentration risk arising from obligors and from counterparties of trading transactions. Furthermore, there can be concentrations in collateral instruments or protection sellers. *Market risk* can also contain concentrations, e.g. if there are high exposures in a specific currency. Concentration in a bank's liabilities can arise in refinancing instruments or refinancing counterparties and depositors. These concentrations can lead to an increased *liquidity risk*. Moreover, there can be risk concentration in the execution or processing of transactions, e.g. if there is a high degree of dependence on a specific IT-system. This is referred to as *operational concentration risk*.[127] As lending is usually the main activity of a bank, the most important type of risk concentration is *credit risk* concentration.[128] Against this background, this type of concentration risk will be analyzed in-depth in the following. In the literature, it is often distinguished between three types of credit concentration risk:

- Name concentration
- Sector concentration
- Credit contagion

[126] BCBS (2005a), § 770.

[127] Cf. Deutsche Bundesbank (2006), p. 36 f.

[128] Cf. BCBS (2005a), § 771.

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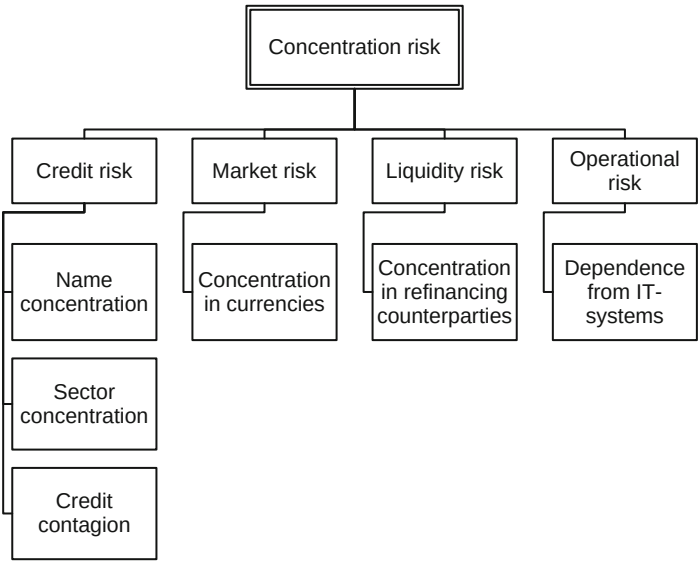

Fig. 3.1 Types of concentration risk. Cf. Deutsche Bundesbank (2006), p. 37

The BCBS distinguishes between two sorts of *name concentration*.[129] One type of concentration risk pertains to an exposure to one firm or to a conglomerate of economically highly dependent firms[130] that is extremely large compared to the rest of the exposures of the portfolio. In such a situation the default risk of the portfolio is significantly driven by the idiosyncratic risk of this individual debtor. This type of concentration will be called "individual name concentration". The second type of name concentration occurs if the bank holds a portfolio containing a relatively small number of firms, each of them with large exposures. Such a portfolio is hardly diversified because of the quite small number of debtors. Thus, a bank faces high losses if several defaults appear, even if they occur accidentally and are not driven by default correlation of the firms. This type of concentration can be denoted as "portfolio name concentration".

The term *sector concentration* refers to significant exposures to groups of counterparts whose likelihood of default is driven by common underlying factors, such as industry sectors or geographical locations.[131] Even if the modeling of these types of sectors is usually similar, the concentrations themselves have some different characteristics. Industry concentrations are mainly related to corporate loans, which have a higher PD if the industry sector is in an economic downturn.

[129] See BCBS (2005b, c).

[130] Under Basel II such a conglomerate is called "connected group"; see BCBS (2005a), § 423.

[131] In a document about technical aspects of the management of concentration risk of the CEBS, examples of common risk factors that possibly lead to sector concentrations also include currencies and credit risk mitigation measures; cf. CEBS (2006), § 25.

In principle, the same is true for geographical concentrations but this type of concentration is also relevant for retail loans and sovereigns. Furthermore, geographical concentration risk includes not only regional but also country risk, which covers different risk categories such as political and transfer risk.[132]

The third type of concentration risk is *credit contagion*. In many cases, two or more companies have a business connection that increases the joint probability of default. This connection is often asymmetric so that a default of firm 1 leads to an increased PD of firm 2 whereas a default of firm 2 shows only minor effects on the PD of firm 1. If the connection is very strong, the firms have to be merged to one connected group. In all other cases only the weaker connection to the overall sector is accounted for, which leads to an underestimation of the true risk.[133] Therefore, credit contagion is in a way between name and sector concentration risk. A typical "micro-structural channel" for this type of concentration risk is the interbank lending market, where a default of one bank could trigger a default of other banks, especially if the loans are uncollateralized and uninsured. Furthermore, suppliers and buyers of goods are often linked via trade credits.[134]

3.2 Incurrence and Relevance of Concentration Risk

Although the expression "concentration risk" expresses the negative aspect of concentration, this does not necessarily mean that it is worthwhile to implement a diversification strategy. As concentration usually stems from *specialization*, a bank can have significant informational advantages in its area of specialization. For example, a bank with a portfolio consisting only of a small number of obligors contains high name concentration but typically knows its obligors very well and can therefore evaluate the firm-specific situation better than others. A bank which is specialized on several industry sectors or geographical locations can have specific knowledge of the relevant markets and the economic environment. As a consequence, in principle a specialized bank could use its informational advantage to generate higher returns and/or lower risk.

In the literature, there exist contradictory statements whether diversification of an intermediary is risk decreasing or increasing and whether diversification increases or decreases the firm value. In *neoclassical economics*, diversification is clearly risk reducing given a constant expected return if the asset returns are

[132]Cf. Deutsche Bundesbank (2006), p. 43.

[133]If the connected companies have symmetric dependencies, it would also be possible to build a new sector with a high correlation inside of the sector. However, in practical implementations the sectors are often constructed with respect to geographical regions or industry branches so that the sector factors can be interpreted as macro-economic factors. Hence, the risk stemming from a connection of firms is usually not covered, regardless of whether the connection is symmetric or asymmetric.

[134]Cf. Giesecke and Weber (2006), p. 742.

non-perfectly correlated, which was shown by Markowitz (1952, 1959). Nevertheless, if there do not exist any market imperfections, there is no advantage of a bank's diversification because the diversification could also be done by private investors. Moreover, financial intermediaries would not even exist in the context of the assumed perfect market. An approach which is more suitable for explanation of this strategic decision is the *principal agent theory*. For instance, according to the fundamental work of Diamond (1984), the main task of financial intermediaries is "delegated monitoring" of the obligors, which leads to a reduction of monitoring costs compared to direct investments without an intermediary. Furthermore, the monitoring costs decrease with higher diversification, which directly leads to the result that diversification is advantageous.[135] By contrast, it is often argued that any firm – financial institution or other – should be specialized on a single business line in order to benefit from the management's expertise, whereas diversification should be done by the investors (see Berger and Ofek 1996; Servaes 1996; Denis et al. 1997). In the theoretical model of Winton (1999), several aspects of diversification are addressed: It is assumed that a bank that diversifies into new sectors faces the problem of adverse selection if established banks are already active in the new sectors; this leads to negative consequences on risk and return. Furthermore, monitoring incentives are usually lower when a bank is diversified, leading to a risk-shifting problem. Altogether, even if diversification leads to a smaller impact of downturns in single sectors, it is mostly risk increasing.[136] Empirical studies largely indicate that diversified banks incorporate higher risk and often at the same time lower returns (see Demsetz and Strahan 1997; Acharya et al. 2006; Deng et al. 2007). Furthermore, according to DeLong (2001) only bank mergers which are focused with respect to the dimensions of activity and geography create value. These results are widely in line with the model of Winton (1999).

Relying on the advantages of specialization, the business models of several financial institutions imply a high degree of concentration, like savings banks and credit cooperatives, which are usually regionally focused, and building societies or automotive financial services providers, which are specialized on specific products. Also a combination of both regional and industry expertise is observable, e.g. the HSH Nordbank is the world's largest ship financier but also regionally focused on Germany's North Sea and Baltic Sea coasts and the Stadtsparkasse Köln is a regional savings bank that is specialized on the German media industry.[137]

[135]The monitoring costs are independent of investment size, thus if the monitoring is delegated to an intermediary by many investors, these costs can be reduced. Of course, now the intermediary itself has to be monitored. As state verification is only necessary in case of (imminent) default and the PD of the bank is lower than of the single investments – this is due to diversification of the bank – the monitoring costs of the bank are relatively low leading to the mentioned results.

[136]The model results in a negative effect of diversification on the risk of a bank if the loans in a bank's home sector have high or low PDs. Only in the case of medium default probabilities of the loans in the home sector, diversification can lead to a risk reduction.

[137]Cf. Kamp et al. (2005), p. 1.

The mentioned advantages of specialization are sometimes cited as evidence that there is no additional risk stemming from concentration and therefore there is no additional need for economic capital. Even if some aspects of this argument are comprehensible, it does not hold in general. To begin with, a concentrated bank does not necessarily have informational advantages over other banks. Firstly, it is a necessary condition that the concentration is a result of expertise in the sector. If this is not fulfilled, the concentration is risk increasing only. As mentioned before, in some sectors there might be a multitude of banks with expertise so that the degree of competition highly influences the risk and return. Therefore, a bank must be better in the generation and procession of information than competitors to earn a specialization premium and not to be faced with adverse selection. This point is especially challenging for globally relevant industries as the bank must compete with other financiers worldwide. If a bank has the ability to benefit from specialization, this advantage has to be used not only to increase the return but also to reduce the risk. For example, many venture capital firms or hedge funds have significant industry expertise but do not have a reputation for their risk-averse investments. Moreover, in empirical studies indicating the benefit of specialization, the risk is often measured in terms of volatility.[138] However, as a consequence of non-normality of the portfolio loss distribution, this does not assure that the risk measures which are relevant for economic capital calculations, e.g. the VaR and the ES, are reduced as well. This can be illustrated very intuitively: A bank which is the global market leader in financing of airplanes and of machine tools might be capable of differentiating between risky and less risky lending activities in these areas and uses this knowledge to hold a portfolio with high quality and low volatility. Now assume that one or both of these sectors are faced with a material drop in demand, so there is a sector-specific downturn. Even if the bank perceived some early-warning indicators and was able to reduce the investment in these sectors, there is a high probability that the institution will suffer substantial losses. Thus, in a worst-case scenario, which is relevant for the determination of the economic capital requirement, it is reasonable to assume that many concentrated portfolios are more vulnerable than non-concentrated portfolios. To sum up, there are good arguments that a bank can benefit from specialization in terms of an increased risk/return ratio. But if it has to be assured that the bank survives economic downturn scenarios (with high probability), it should hold an additional capital buffer.

The practical relevance of this issue can be seen from many bank failures or even banking crises that resulted from or at least in combination with concentration risk. During the 1980s and 1990s, more than 1,000 savings and loan associations defaulted in the United States in the savings and loan crisis. Although the problem cannot be reduced to sectoral concentrations, "the banking problems of the 1980s and 1990s came primarily [...] from unsound real estate lending"[139] with a

[138] Cf. Acharya et al. (2006) and Behr et al. (2007).

[139] Seidman et al. (1997), p. 57.

significant increased share of this type of lending.[140] In Scandinavia, the real estate crisis of the early 1990s also led to many bank defaults.[141] The high concentrations in structurally lagging regions led to a high degree of non-performing loans and finally to the divestiture of the Schmidt-Bank in 2001. Also the ongoing financial crisis has its seeds in lax real estate lending, in this case mainly to creditors with low creditworthiness and without down-payment in the United States (subprime lending). A huge amount of the exposure was transferred worldwide to institutional investors via structured financial products, mainly residential mortgage backed securities. These products showed a material price drop, which was due to an underestimation of the correlation between default rates and especially between the residential mortgages. Thus, the concentration risk in the collateral pool was underestimated.[142] In BCBS (2004a) several additional examples of banking crises are studied and a high proportion is found to be affected by risk concentrations.[143]

3.3 Measurement and Management of Concentration Risk

As mentioned in the introductory statement, the Basel Committee on Banking Supervision already recognized the high importance of credit risk concentrations in the Basel framework: *"Risk concentrations are arguably the single most important cause of major problems in banks"*.[144] Against this background, it seems necessary to account for concentration risk in the banks' minimum capital requirements.[145] However, the quantitative framework in Pillar 1 of Basel II does not account for concentration risk at all, since it is based on the ASRF framework, which assumes that (A) the portfolio is infinitely fine-grained and (B) only a single systematic risk factor influences the credit risk of all loans in the portfolio. Thus, the first assumption implies that there is no name concentration in the portfolio, which means that all idiosyncratic risk is diversified completely. The second assumption implicates that there exists no sector concentration such as industry-specific or geographical risk concentrations and also no credit contagion. These are

[140]Prior to the 1980s, less than 10% of U.S. bank portfolios were invested in real estate portfolios, whereas by the mid-1980s some banks increased this share to 50 or 60%; cf. Seidman et al. (1997), p. 58.

[141]Cf. Deutsche Bundesbank (2006), p. 38.

[142]"Structured Finance CDO enhancement levels were not commensurate with the higher observed correlations in the performance of collateral assets during stressed market conditions, particularly for portfolios with elevated risk concentrations or exposure to a narrow, common set of risk factors"; see Hansen et al. (2009), p. 4.

[143]Credit concentration risk, mostly in real estate, is cited to be a relevant cause of bank failures in nine out of the thirteen episodes; cf. BCBS (2004b), p. 66 f.

[144]See BCBS (2005a), § 770.

[145]The term concentration risk will be referred to concentrations in lending if not indicated elsewise.

idealizations that can be problematic for real-world portfolios. But since it is difficult to incorporate credit risk concentrations in analytic approaches and since there is not yet an approach which is widely accepted as the industries "best practice", in Basel II there is no quantitative approach mentioned how to deal with risk concentrations.[146] Instead, it is only qualitatively demanded in Pillar 2 of Basel II that "*Banks should have in place effective internal policies, systems and controls to identify, measure, monitor, and control their credit risk concentrations*".[147] Thus, it is each bank's task to decide how to meet these requirements concretely. However, since this topic is very important for the stability of the banking system, several supervisory documents regarding this issue have been published that analyze the state of the art and give guidance for institutions and supervisors. The Basel Committee launched the "Research Task Force Concentration Risk", which has presented its final report in BCBS (2006). The report contains information about the state of the art in current practice and academic literature, an analysis of the impact of departures from the ASRF model and a review of some methodologies to measure name and sector concentrations. An additional workstream has focused on stress testing against the background of risk concentrations. In 1999 the Joint Forum[148] published "Risk Concentrations Principles" to ensure the prudent management and control of risk concentrations in financial conglomerates through the regulatory and supervisory process. Joint Forum (2008) analyzes the progress of financial conglomerates in identifying, measuring, and managing risk concentrations on a firm-wide basis and across the major risks to which the firm is exposed. Furthermore, the document surveys the current regulatory requirements (quantitative and qualitative) in the European Union, the United States, Japan, and Canada.[149] In CEBS (2006), the Committee of European Banking Supervisors published a survey on current market practices for the identification and measurement of concentration risk. Moreover, five principles for institutions and six principles for supervisors are given as guidance for the treatment of concentration risk under Pillar 2, which specifies the Capital Requirements Directive (CRD) of the European Union regarding concentration risk (see Table 3.1).

[146]Until the second consultative document a version of the so-called granularity adjustment was part of Basel II for measuring name concentrations, but because of some theoretical shortcomings and as it appeared to be too complex for many banks it was cancelled in the final Basel capital rules. The effectiveness and the eligibility of the (cancellation of the) granularity add-on from the second to the third consultative document of Basel II is only discussed vaguely in the literature; see e.g. Bank and Lawrenz (2003), p. 543.

[147]See BCBS (2005a) § 773.

[148]The Joint Forum was established in 1996 under the aegis of the Basel Committee on Banking Supervision (BCBS), the International Organization of Securities Commissions (IOSCO) and the International Association of Insurance Supervisors (IAIS) to deal with issues common to the banking, securities and insurance sectors, including the regulation of financial conglomerates. The Joint Forum is comprised of an equal number of senior bank, insurance and securities supervisors representing each supervisory constituency. See Joint Forum (2009).

[149]Cf. Joint Forum (2008), p. 35 ff.

Table 3.1 Guidance for institutions and supervisors considering concentration risk. See CEBS (2006), p. 11 ff

Guidance for Institutions	
Concentration 1	All institutions should have clear policies and key procedures ultimately approved by the management body in relation to exposure to concentration risk
Concentration 2	In application of Article 22 of the Capital Requirements Directive, institutions should have appropriate internal processes to identify, manage, monitor, and report concentration risk which are suitable to the nature, scale and complexity of their business[a]
Concentration 3	Institutions should use internal limits, thresholds or similar concepts, as appropriate, having regard to their overall risk management and measurement
Concentration 4	Institutions should have adequate arrangements in place for actively monitoring, managing, and mitigating concentration risk against agreed policies and limits, thresholds, or similar concepts
Concentration 5	Institutions should assess the amount of internal capital which they consider to be adequate to hold against the level of concentration risk in their portfolio
Guidance to Supervisors	
Concentration 6	Supervisors will collect sufficient information from institutions on which to base their assessment
Concentration 7	The scope of application of the supervisors' assessment of concentration risk is that used for the Supervisory Review Process (SRP)
Concentration 8	Supervisors will use quantitative indicators, where appropriate, within their Risk Assessment Systems to assess degrees of concentration risk
Concentration 9	The supervisory review should encompass not only quantitative aspects but also the qualitative and organizational aspects of concentration risk management
Concentration 10	Supervisors can draw on stress tests performed by institutions to assess the impact of specific economic scenarios on concentrated portfolios
Concentration 11	Supervisors will pay particular attention to those institutions which are highly concentrated by customer type or specialized nature of product

[a]The Article 22 of the CRD says that every credit institution requires "effective processes to identify, manage, monitor, and report the risks it is or might be exposed to"

As can be seen from these principles, there is a variety of actions that should be taken to handle concentration risk. Due to *Principle* 1, the management body of a credit institution shall define clear policies and procedures regarding concentration risk, which may depend on the business strategy and the risk appetite of the bank. Furthermore, banks should identify and measure concentration risk, which is a necessary condition for managing and monitoring these risks (*Principle* 2). The identification and measurement of concentration risk can be based on rather heuristic or analytical approaches. For example, a bank could measure the size of the top "x" largest exposures or connected exposures relative to the relevant numeraire (e.g. the balance sheet amount). This quantification could also be applied to industry sectors, geographical regions, or product lines.

Moreover, the concentration could be quantified with Gini coefficients or the Herfindahl–Hirschmann Index, which will be described in Sect. 3.4. A review of the literature regarding model-based approaches for the measurement of concentration

risk will be given in Sect. 3.5.[150] Principles 3–5 can be seen as further requirements regarding the monitoring and management of concentration risk, which is already demanded in Principle 2. One rather simple but effective action for this purpose is to establish an internal limit system, which shall prevent from undesirable high concentrations in large individual or connected exposures, industry sectors, geographical regions, or product lines (*Principle* 3). A starting point for the limit on large individual exposures can be the directive of the European Union, which demands that a large individual exposure may not exceed 25% of a credit institutions own funds.[151] However, the internal limit system should set additional limits, which are in line with the degree of risk taking that is accepted by the management body. These limits can be based on the aforementioned measurement techniques. Of course, a bank should also have arrangements in place, which actions shall be taken if risk concentrations are detected that are problematic concerning the risk policy or limit system (*Principle* 4). These actions will usually start with a more detailed review of the concerned exposure. Furthermore, stress tests and scenario analyses can be applied. Depending on the results of the analyses, several mitigating actions can be applied, which range from rather passive to active actions. Possible consequences are the modification of concentration limits, the allocation of additional internal capital, a transfer of credit risk to third parties, e.g. using credit derivatives,[152] collateral, or guarantees, and an adjustment of new business acquisitions in order to revert to a lower concentration level. Regardless of whether risk concentrations were originally intended by the bank or not (as it may be the case mentioned in Principle 4), the bank should assess an adequate amount of internal capital against their risk concentrations, which depends on the degree of concentration risk (*Principle* 5). In this context, the onus to demonstrate the adequacy of internal capital will usually be greater for institutions with more concentrated credit portfolios (see also Principle 11). *Principles* 6–11 describe a general guideline for supervisors and advise which actions should be taken during the Supervisory Review Process under Pillar 2 regarding concentration risks.[153] Especially, institutions will be required by supervisors "to show that their internal capital, where considered necessary, is commensurate with the level of concentration risk."[154] This requirement illustrates that from a regulatory perspective the most important issue is the adequate assessment of capital required in Principle 5.

[150]Some additional suggestions are given in CEBS (2006).

[151]Cf. EU (2006), Title 5, Chap. 2, Sect. 5, Article 111.1 [Directive 2006/48/EC].

[152]Large exposures will typically be transferred with credit default swaps (CDS), whereas concentrations in sectors or product lines will often be reduced with collateralized debt obligations (CDO). For a short introduction to CDS, especially regarding modeling purposes, see Bluhm et al. (2003). A description of CDOs and analyses of CDOs against the background of asymmetric information between protection seller and protection buyer are given in Gürtler et al. (2008b), whereas Bluhm et al. (2003) as well as Bluhm and Overbeck (2007) present a good overview for modeling CDOs.

[153]Cf. CEBS (2006).

[154]CEBS (2006), p. 2.

The basis for a meaningful monitoring and management of concentration risk is the proper measurement of these risks (for establishing a limit system, for deciding on the quantity of credit derivative instruments, for allocation of internal capital and so on). Against this background, the focus of this work will be the measurement of concentration risk as well as the resulting assessment of the required capital amount. When measuring concentration risk, it is important to notice the popular different interpretations of concentration risk by banks and supervisors. While this is generally unproblematic for internal policies, it is essential for the allocation of additional capital against concentration risk. Banks often only look at one side of concentration risk – the diversification effect. They argue that the Pillar 1 capital requirement does not measure benefits from diversification. Therefore it is argued that this framework is the non-diversified benchmark and thus the upper barrier for the true capital requirement. On the contrary, supervisors interpret concentration risk as "a positive or negative deviation from Pillar 1 minimum capital requirements derived by a framework that does not account explicitly for concentration risk".[155] The latter perception is justified by the fact that the Pillar 1 capital rules were calibrated on well-diversified portfolios with low name and low sector concentration risk.[156] Thus, if a portfolio is lowly diversified, the risk will be underestimated when using the Basel formula. Therefore additional capital is required to

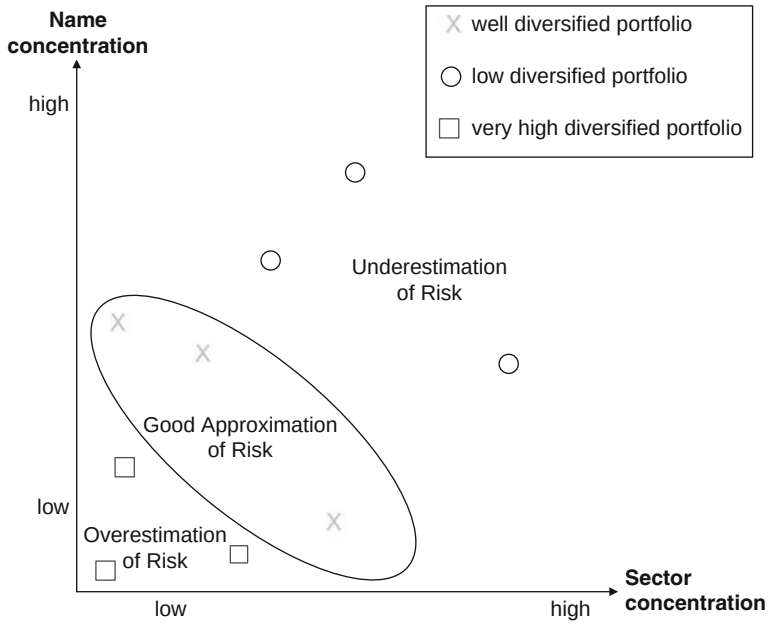

Fig. 3.2 Accuracy of the Pillar 1 capital requirements considering risk concentrations

[155] See BCBS (2006), p. 7.

[156] Cf. BCBS (2006), p.14, and CEBS (2006), § 18.

capture these types of concentration risk. However, if the portfolio is very highly diversified, the Basel formula can overestimate the "true" risk. For well-diversified portfolios, the Basel formula is a good approximation of the "true" risk. This relation is highlighted in Fig. 3.2.

As noticed above, for a quantification of concentration risk there exist some heuristic and some analytical approaches in the literature. Both will be presented in the following sections.

3.4 Heuristic Approaches for the Measurement of Concentration Risk

The most common heuristic approaches quantify concentration risk with the Gini coefficient or the Herfindahl–Hirschmann Index.[157] In principle, both measures can be applied to name concentrations and sector concentrations as well. For a description of the Gini coefficient it is helpful to introduce the *Lorenz curve* first. The Lorenz curve is a graphical representation of the distribution of a variable z and the degree of inequality of this variable. For discrete variables, the Lorenz curve is the piecewise linear function connecting the points (x_i, y_i) with

$$x_i = \frac{i}{n} \quad \text{and} \quad y_i = \frac{\sum_{j=1}^{i} z_{j:n}}{\sum_{j=1}^{n} z_j}, \tag{3.1}$$

where $z_{j:n}$ is the order statistics of z, so that elements of z are sorted into an increasing order.[158] Thus, y_i is the relative amount of the i smallest elements of z_i, and x_i is the relative amount of included elements. For example one point on the Lorenz curve could show that the smallest 20% elements of a variable account for 5% of the total amount.[159] If the elements are of equal size, the function is simply $y = x$, which is called the "line of perfect equality". The opposite, the "line of perfect inequality" is a situation, where one element accounts for the total amount of the variable so that $y = 0$ for all $x < 1$ and $y = 1$ if $x = 1$. Against the background of name concentrations, the variable z could be identified with credit exposures. Thus, the Lorenz curve shows the cumulative share of exposures for

[157] Cf. Deutsche Bundesbank (2006), p. 40 ff., and BCBS (2006), p. 8 ff.

[158] Cf. Sect. 2.2.4.

[159] A common example for the usage of the Lorenz curve is the concentration analysis of income distributions.

each cumulative share of credits.[160] As the relative share of an exposure is defined by the weight w_i, the expression (3.1) simplifies to

$$x_i = \frac{i}{n} \quad \text{and} \quad y_i = \sum_{j=1}^{i} w_{j:n}. \tag{3.2}$$

Fig. 3.3 exemplarily shows the Lorenz curve for credit exposures. The closer the curve is to the diagonal line, the smaller are inequality and concentration of the exposures.

The Lorenz curve is directly related to the *Gini coefficient*, which expresses the degree of inequality in a single number between 0 (perfect equality) and 1 (perfect inequality). As area A between the diagonal line and the Lorenz curve reflects the degree of inequality, the Gini coefficient G is defined as twice this area so that the area is transformed from $A \in [0,\ 0.5]$ to $G \in [0,\ 1]$. The area under the Lorenz curve can be calculated as a sum of trapezoids, leading to a Gini coefficient of

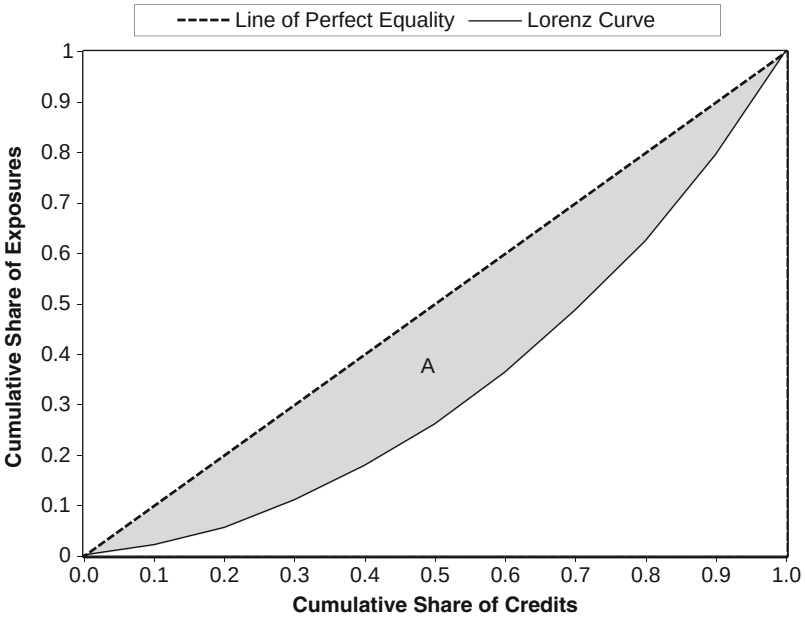

Fig. 3.3 Lorenz curve for credit exposures

[160] In many cases it makes sense to aggregate all credit exposures of one obligor to one exposure before. E.g. in corporate portfolios a default is usually referred to the obligor such that all credits are in default if the obligor is past due more than 90 days on any (material) credit obligation. On the contrary, in retail portfolios the defaults can be handled on contract instead of obligor level. In this case the credits can be handled separately.

$$\begin{aligned} G &= 2 \cdot A = 2 \cdot \left(\frac{1}{2} - \sum_{i=1}^{n} \text{Trapezoid}_i \right) \\ &= 2 \cdot \left(\frac{1}{2} - \sum_{i=1}^{n} \frac{1}{2} \cdot (x_i - x_{i-1}) \cdot (y_i + y_{i-1}) \right) \\ &= 1 - \sum_{i=1}^{n} (x_i - x_{i-1}) \cdot (y_i + y_{i-1}). \end{aligned} \tag{3.3}$$

The advantage of the Lorenz curve and the Gini coefficient is that they can easily be implemented and interpreted. However, there are several disadvantages that delimit the benefit. One problem is that the results do not account for the number of credits and therefore for no portfolio name concentration. For example, a poorly diversified portfolio consisting of two credits with exposure weights $w_1 = 0.3$ and $w_2 = 0.7$ has a Gini coefficient of $G = 1 - (0.5 \cdot 0.3 + 0.5 \cdot 0.7) = 0.5$ and the corresponding Lorenz curve is defined by x_i and y_i from (3.2). A portfolio with significantly lower name concentration could be constructed by dividing each of the credits in 100 credits with equal weight, but this portfolio still has the identical Lorenz curve and a Gini coefficient of $G = 0.5$ since the degree of inequality remains identically. Thus, only individual name concentration can be expressed by this method but no portfolio name concentration. Another problem is that no correlation effects and no different portfolio qualities can be accounted for. Two portfolios with identical exposure distributions but different correlation or PD structures have the same Lorenz curve but different name concentrations.

The Lorenz curve and the Gini coefficient can also be applied to sector concentrations. For this purpose, the exposures of each industry sector or each geographical region could be aggregated so that the concentration regarding the exposure size of sectors is measured. The problem that the number of sectors is not accounted for is less problematic because the number of sectors is usually fixed for a single bank. Even if the Lorenz curve is not comparable between different banks due to a different sector definition, the variation of the Lorenz curve in time can be observed for a single bank. However, the problem regarding correlation effects is very critical for sector concentrations, as there is no chance to distinguish between a "diversification" across highly dependent or marginally related sectors.

The *Herfindahl–Hirschmann Index* (HHI) is another measure, which is often used for a quantification of concentrations. As already mentioned in Sect. 2.6, the HHI is defined as the sum of squared weights of elements (exposures) and the reciprocal is the effective number of elements (exposures):

$$HHI = \sum_{i=1}^{n} {w_i}^2 = \frac{1}{n^*}. \tag{3.4}$$

In comparison to the Gini coefficient, the advantage of the HHI is that the index accounts for the number of credits, which is relevant for portfolio name

concentration. In the example above, the HHI is 0.58 for the two-credit-case and 0.0058 for the case of dividing each of these credits into 100 equal sized credits. Moreover, there is a weak theoretical link between the HHI and name concentration risk as a HHI of zero is a necessary condition for infinite granularity.[161] Hence, the HHI seems to be a better measure of name concentration than the Gini coefficient. As an ad-hoc measure of sector concentration the HHI faces the problems of neglecting the correlation and PD structure, too. Thus, this index can only provide a superficial estimate of sectoral concentrations.

Against this background, the mentioned heuristic approaches should only be used for a rough impression of the degree of concentration in the portfolio and of the variation of concentration in time. Since none of the methods is capable of including correlation effects, which are a core element of concentration risk, and no information about the capital requirement can be achieved, it appears necessary to additionally measure concentration risk with more sophisticated, model-based approaches.

3.5 Review of the Literature on Model-Based Approaches of Concentration Risk Measurement

As noticed in Sect. 3.2, *name concentrations* can be divided into individual name concentrations and portfolio name concentrations. The latter type of name concentrations can be analytically approximated with the so-called granularity adjustment. The idea of the adjustment is based on Gordy (2001), who finds that the add-on for undiversified risk is almost linear in terms of the reciprocal of the number of credits $1/n$ and estimates the slope of the term by simulation based on the CreditRisk$^+$ model. Wilde (2001) derives the granularity adjustment formula analytically by linear approximations around the VaR resulting from the ASRF model. He shows that the formula implemented in the second consultative paper (CP2) of Basel II only leads to convincing results in a CreditRisk$^+$ model but differs from the theoretically derived results when the adjustment formula is calibrated consistent with the Vasicek model. The adjustment formula has been improved by Pykhtin and Dev (2002) so that it is valid for a broader range of PDs. Gordy (2003) generalizes the adjustment formula and numerically analyzes the accuracy of the granularity adjustment when applied to the CreditRisk$^+$ model for several portfolios. Martin and Wilde (2002), Rau-Bredow (2002) and Gordy (2004) obtain the granularity adjustment using a more straightforward approach on the basis of a Taylor series expansion, applying the results of Gouriéroux et al. (2000) for the first two derivatives of the VaR. Using higher derivatives of VaR derived by Wilde (2003), Gürtler et al. (2008a) extend the adjustment to terms of

[161] Cf. (2.86).

higher order to improve the accuracy. Furthermore, they numerically analyze the impact of unsystematic credit risk and the accuracy of the granularity adjustment when applied to the Vasicek model in detail. While these articles use the VaR as the risk measure, Pykhtin (2004) and Rau-Bredow (2005) derive the granularity adjustment for the case of ES being the relevant risk measure. An approach related to Wilde (2001) is the granularity adjustment from Gordy and Lütkebohmert (2007). Their formulas need less data than the original granularity adjustment but are based on the CreditRisk$^+$ model and not on the Vasicek model, which the IRB Approach is based on. In contrast to these approaches, Emmer and Tasche (2005) refer to individual name concentrations. They assume that one single obligor accounts for a significant share of the overall portfolio, while the rest of the portfolio remains infinitely granular. That is why it is called a semi-asymptotic approach.

There also exist a few analytic and semi-analytic approaches that account for *sector concentrations*. One rigorous analytical approach is Pykhtin (2004), which is based on a similar principle as in Martin and Wilde (2002), expanding the Taylor series expansion to a multi-factor context. This multi-factor adjustment is applied to both the VaR and the ES. An alternative is the semi-analytic model from Cespedes et al. (2006). The authors determine a formula that transforms the VaR of the IRB Approach into a multi-factor approximation of the VaR through a complex numerical mapping procedure. Düllmann (2006) extends the binomial extension technique (BET) model from Moody's by incorporating the "infection probability" of Davis and Lo (2001). This additional parameter has been calibrated in a way that the VaR of a multi-factor model is approximated. Based on the principles of Emmer and Tasche (2005), Tasche (2006b) suggests an extension of the ASRF framework towards an asymptotic multi-risk-factor setting. Some numerical work on the performance of the Pykhtin model has been done by Düllmann and Masschelein (2007). Furthermore, Düllmann (2007) presents a first comparison of different approaches on sector concentration risk. Gürtler et al. (2010) adjust the models of Pykhtin (2004) and Cespedes et al. (2006) to be consistent with the IRB Approach. Furthermore, they compare the performance of the models on the basis of a simulation study.

One of the first contributions to the literature that models *credit contagion* is Davis and Lo (2001). In their model, the authors distinguish between direct defaults and indirect defaults, which occur through an infection from directly defaulting firms. Hammarlid (2004) shows how independent sectors can be aggregated within the model of Davis and Lo (2001). Giesecke and Weber (2006) model the probability of financial distress depending on the number of financially distressed business partners in a reduced-form model. However, these contributions assume homogeneous credits – for Hammarlid (2004) at least inside the independent sectors – and a symmetric dependence structure. Neu and Kühn (2004) and Egloff et al. (2007) allow for more realistic credit portfolios consisting of credits with heterogeneous characteristics and asymmetric dependence structures but the computation of loss distributions needs Monte Carlo simulations. Neu and Kühn (2004) is based on a multi-factor default-mode model. The authors add a term to the

individual asset return that leads to an increased PD if connected firms are in financial distress and to a decreased PD if competitors default. Egloff et al. (2007) extend a multi-factor model, which allows for rating migrations, with asymmetric microstructural dependencies. In contrast to Neu and Kühn (2004), there is no additional term in the asset return but the idiosyncratic component is divided into a "true" unsystematic fraction and a fraction that is influenced by defaults of related firms.